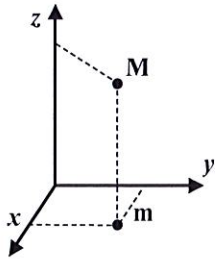


Formulaire d'analyse vectorielle

Calcul vectoriel

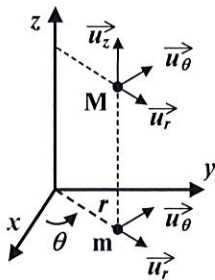
$$\begin{array}{l|l}
 \overrightarrow{\text{rot}} \overrightarrow{\text{grad}} V & = \vec{0} \\
 \text{div} \overrightarrow{\text{rot}} \vec{A} & = 0 \\
 \text{div} \overrightarrow{\text{grad}} V & = \Delta V \\
 \overrightarrow{\text{rot}} \overrightarrow{\text{rot}} \vec{A} & = \overrightarrow{\text{grad}} \text{div} \vec{A} - \Delta \vec{A}
 \end{array}
 \quad \left| \quad
 \begin{array}{l}
 \overrightarrow{\text{grad}}(V_1 V_2) & = V_1 \overrightarrow{\text{grad}} V_2 + V_2 \overrightarrow{\text{grad}} V_1 \\
 \overrightarrow{\text{rot}}(V \vec{A}) & = V \overrightarrow{\text{rot}} \vec{A} + \overrightarrow{\text{grad}} V \wedge \vec{A} \\
 \text{div}(V \vec{A}) & = V \text{div} \vec{A} + \overrightarrow{\text{grad}} V \cdot \vec{A} \\
 \text{div}(\vec{A}_1 \wedge \vec{A}_2) & = \vec{A}_2 \cdot \overrightarrow{\text{rot}} \vec{A}_1 - \vec{A}_1 \cdot \overrightarrow{\text{rot}} \vec{A}_2
 \end{array}$$

Coordonnées cartésiennes



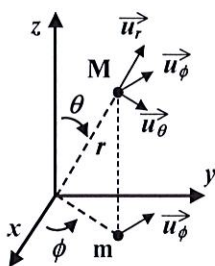
$$\begin{aligned}
 \overrightarrow{\text{grad}} V &= \frac{\partial V}{\partial x} \vec{u}_x + \frac{\partial V}{\partial y} \vec{u}_y + \frac{\partial V}{\partial z} \vec{u}_z \\
 \text{div} \vec{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\
 \overrightarrow{\text{rot}} \vec{A} &= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \vec{u}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \vec{u}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \vec{u}_z \\
 \Delta V &= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \quad \Delta \vec{A} = \Delta A_x \vec{u}_x + \Delta A_y \vec{u}_y + \Delta A_z \vec{u}_z
 \end{aligned}$$

Coordonnées cylindriques



$$\begin{aligned}
 \overrightarrow{\text{grad}} V &= \frac{\partial V}{\partial r} \vec{u}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \vec{u}_\theta + \frac{\partial V}{\partial z} \vec{u}_z \\
 \text{div} \vec{A} &= \frac{1}{r} \frac{\partial r A_r}{\partial r} + \frac{1}{r} \frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z} \\
 \overrightarrow{\text{rot}} \vec{A} &= \left(\frac{1}{r} \frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z} \right) \vec{u}_r + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) \vec{u}_\theta + \frac{1}{r} \left(\frac{\partial r A_\theta}{\partial r} - \frac{\partial A_r}{\partial \theta} \right) \vec{u}_z \\
 \Delta V &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2}
 \end{aligned}$$

Coordonnées sphériques



$$\begin{aligned}
 \overrightarrow{\text{grad}} V &= \frac{\partial V}{\partial r} \vec{u}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \vec{u}_\theta + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \vec{u}_\phi \\
 \text{div} \vec{A} &= \frac{1}{r^2} \frac{\partial r^2 A_r}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial \sin \theta A_\theta}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi} \\
 \overrightarrow{\text{rot}} \vec{A} &= \frac{1}{r \sin \theta} \left(\frac{\partial \sin \theta A_\phi}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right) \vec{u}_r + \dots \\
 &\quad \dots + \frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial r A_\phi}{\partial r} \right) \vec{u}_\theta + \frac{1}{r} \left(\frac{\partial r A_\theta}{\partial r} - \frac{\partial A_r}{\partial \theta} \right) \vec{u}_\phi \\
 \Delta V &= \frac{1}{r} \frac{\partial^2 r V}{\partial r^2} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}
 \end{aligned}$$

Théorèmes

Théorème d'Ostrogradsky-Green :

S étant une surface fermée, τ le volume intérieur à S ,

$$\oint_{(S)} \vec{A} \cdot d\vec{S} = \int_{(\tau)} (\text{div} \vec{A}) d\tau$$

Théorème de Stokes-Ampère :

C étant une courbe fermée bordant une surface S ,

$$\oint_{(C)} \vec{A} \cdot d\vec{l} = \int_{(S)} (\overrightarrow{\text{rot}} \vec{A}) \cdot d\vec{S}$$